Top-Down Physics

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Outline

- Physics Education Research
  - Some guiding principles
- The Top-Down Approach
  - From specific to general *and* back…
- Examples
  - Central Forces (Classical Mechanics)
  - Tooth-pick Toss (Computational Physics)
  - MRI Applications (Electricity and Magnetism)
- Implementation notes
  - Available materials

Physics Education Research

- Obstacles to learning (student)
  - Metzer[1] finds *correlation* between learning gain and pre-instruction mathematics skill.
  - Redish[2] et. al., measured a *deterioration* in class expectations (attitude) across semester.
- Measuring our success (teacher)
  - Dukes and Pritchard[3] argue that conceptual understanding is not necessary for quantitative success.

1. D.E. Meltzer, Am. J. Phys. 70 (12), December 2002.
2. E.F. Redish, J.M. Saul, and R.N. Steinberg, Am. J. Phys. 66 (3), 212-224, (1998).
3. P. Dukes & D. Pritchard, Invited paper for proceedings of Physics Education Research Conference (MIT) 2001.

The Top-Down Approach

- A paradigm with purpose
  - Mathematical knowledge
    - Pre-test, evaluate, and define
  - Conceptual understanding
    - Re-test, demonstrate, and refine
- Practical relevance
  - Stress physical significance of solution
  - Test all “parameters” of general solution
  - Explore with “extension” of specific solution

Tried and Tested

- Francis Marion University
  - Classical Mechanics ✓
  - Electricity and Magnetism
  - Quantum Mechanics
  - Computational Physics ✓
- University of South Carolina
  - Neuro Imaging of Cognition ✓
  - Medical Imaging (Spring 2007)

Example – Central Forces

- Astronomy meets computer algebra
  - Start with radial wave equation

$$E=\frac{\mu}{2}\left(\frac{\partial}{\partial t}r(\theta)\right)^2+\frac{L^2}{2\mu r^2}+U(r)\Rightarrow\frac{\partial^2}{\partial\theta^2}\left(\frac{1}{r}\right)+\frac{1}{r}=-\frac{\mu r^2}{L^2}F(r)$$

  - Solve using Maple

```
> findForce:=proc(form::algebraic)
    global r,F;
    # assume r will depend only on theta
    r:=unapply(form,theta):
    diff(1/r(theta),theta$2)+1/r(theta) = -mu/(L^2)*r(theta)^2*f:
    # solve for the force as a function of the angle
    solve(%,f):
    # solve the orbit for theta
    solve(r=r(theta),theta):
    # substitute the angular form into the force solution
    subs(theta=%,%%):
    # simplify the force and "build" the force function
    F:=unapply(collect(simplify(%),r),r):
    print('r(theta)'=r(theta));
    print('F(r)'=F(r));
  end proc:
```

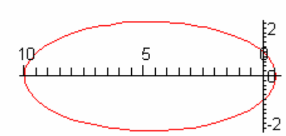
$\Rightarrow$

```
> findForce(alpha/(1+epsilon*cos(theta)));
```

$$r(\theta)=\frac{\alpha}{1+\varepsilon\cos(\theta)}$$

$$F(r)=-\frac{L^2}{\alpha r^2\mu}$$

```
> alpha:=1: L:=1: mu:=1: epsilon:=0.9:
  plot(r(theta),theta=0..2*Pi,
       coords=polar,scaling=constrained);
```



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Central Forces - continued

- Reinforce common sense
  - Uniform linear motion
- Explore “radical” orbits
  - Cardiod motion
- Extend the project
  - Create an animation sequence

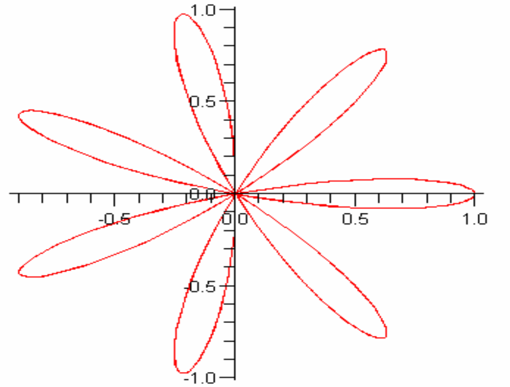
```
> findForce(b/(sin(theta)-m*cos(theta)));
```

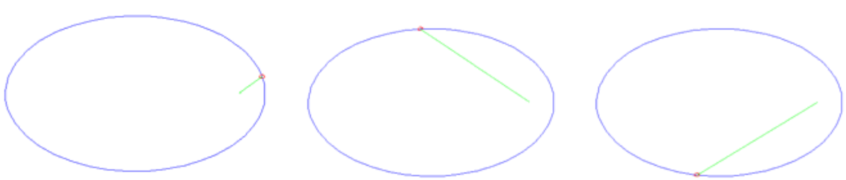
$$r(\theta) = \frac{b}{\sin(\theta) - m\cos(\theta)}$$
$$F(r) = 0$$

```
> findForce(B*cos(n*theta));
```

$$r(\theta) = B\cos(n\,\theta)$$
$$F(r) = \frac{n^2 - 1}{r^3} - \frac{2\,n^2\,B^2}{r^5}$$

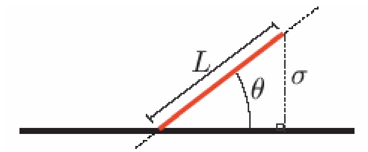
```
> B:=1: n:=7:
  plot(r(theta),theta=0..2*Pi,
       coords=polar,scaling=constrained);
```

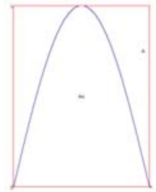




Example – Estimating Pi

- Toothpicks meet technology
  - Ratio of crossing area to total area

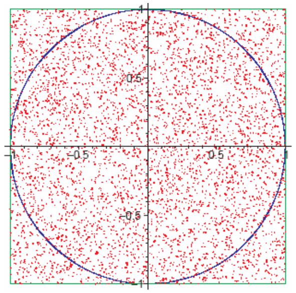


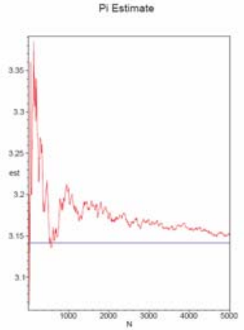


$$\frac{N}{N_c} \propto \frac{A}{A_c} = \frac{\pi L}{\int \sigma(L,\theta)\, d\theta} = \frac{\pi}{2}$$

  - 175 toothpicks yielded ***π*** ≈ 3.21
- Creating a simulation
  - What is essential?

$$\frac{A_{circle}}{A_{square}} = \frac{\pi (l/2)^2}{l^2} = \frac{\pi}{4}$$

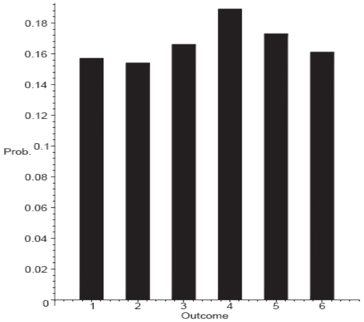




Estimating Pi - continued

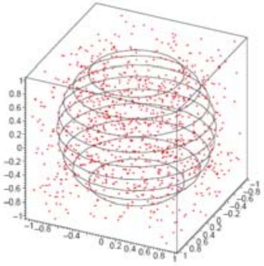
- Refining the analysis
  - Assumption of uniformity

$$\frac{N_{circle}}{N_{square}} \propto \frac{A_{circle}}{A_{square}}$$

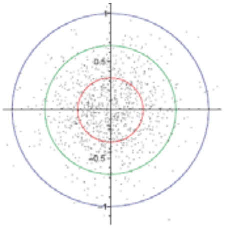


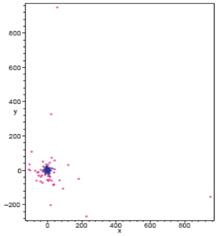
- Extensions
  - Constructing a 3D model

$$\pi \cong \frac{6N_S}{N}$$



  - Different distributions
    - Can they be used here?





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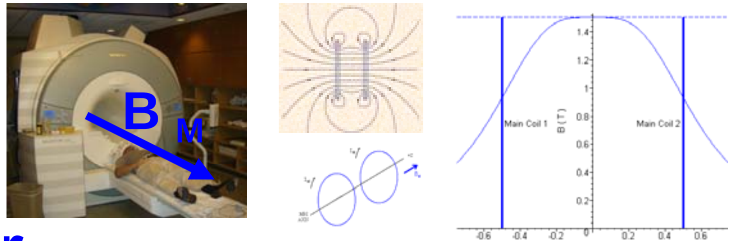
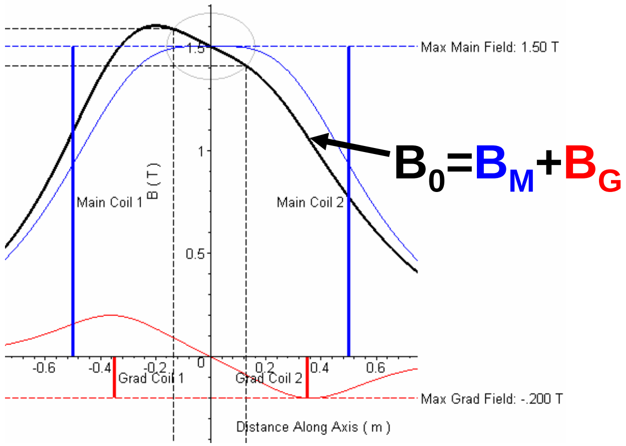
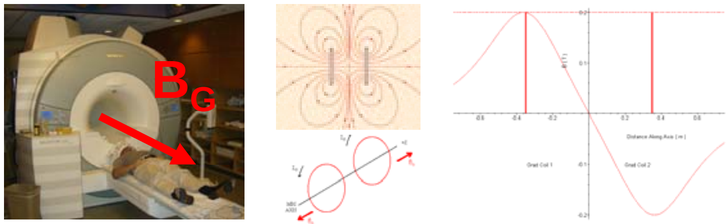
Example – MRI Fields

- Physics meets psychology
- Field Characteristics
  - Main - Helmholtz Coils
  - Gradient – Maxwell Pair



$$B_M = \frac{\mu_0 N_M I_M a^2}{2}\left[\frac{1}{\left(z-\frac{a}{2}\right)^2 + a^2} + \frac{1}{\left(z+\frac{a}{2}\right)^2 + a^2}\right]$$





$$B_G = \frac{\mu_0 N_G I_G b^2}{2}\left[\frac{1}{\left(z-\frac{b}{2}\right)^2 + b^2} - \frac{1}{\left(z+\frac{b}{2}\right)^2 + b^2}\right]$$

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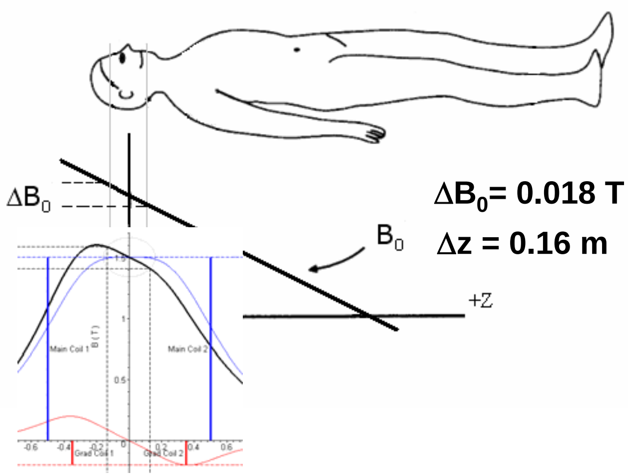
MRI Fields - continued

- Larmor frequency
  - Change in field across sample (ΔB_0)
  - Frequency depends on position (z):

$$\nu_0(z) \cong \frac{\gamma}{2\pi}\left(B_0 + \frac{\Delta B_0}{\Delta z} z\right)$$



- Net Magnetization
  - Field, temp, and material:

$$M = c\frac{B_0}{T} = \frac{1}{V}\frac{\mu_z^2}{k_B}\frac{B_0}{T} \Rightarrow B_{Local} = (1+\chi_m)\mu_0 M = \boxed{\frac{(1+\chi_m)\mu_0\mu_z^2}{Vk_BT}} B_0$$

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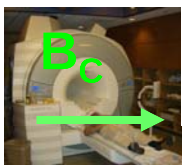
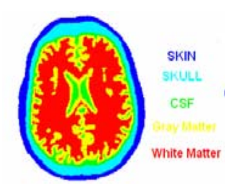
MRI Fields - continued

- Image Formation
  - Integrate to get MRI signal (“k-*space*”)



$$S(t)=\iint_{Area} M_{XY}(x,y,t)\,e^{-i\gamma\int_0^t [xG_X(\tau)+yG_Y(\tau)]dt}\,dxdy$$

  - Gradient selects “Z-slice” to form X-Y image
- Contrast from difference in magnetization
  - *Image at several times (average slices)*



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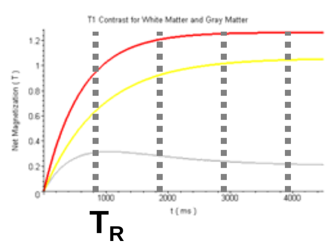

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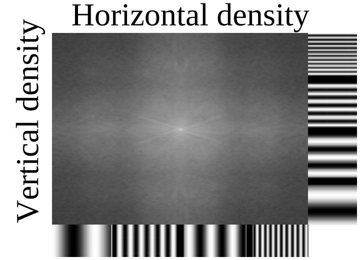

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Implementation Notes

- Obstacles to implementation
  - Tradition!
  - Infrastructure!
  - Time!
- Possible Solutions
  - Combine Classical and E&M material
  - Give group problems in class (add a “lab” day)
  - Test outside of class

Course Materials Available

- Computational Physics
  - Maple and Java source code
  - Two volume text now available
- Applied Physics
  - Maple worksheets and projects
  - New text coming soon
- Site and Contact
  - URL: http://www.evsis.org
  - Email: mjs@sc.edu