

# Top-Down and Free

Top-Down Physics

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# What's in the TDP box?

Top-Down Physics  
Τοπ-Down Physics

Top-Down Physics  
Τοπ-Down Physics  
The Nature of the Physical World

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- Table of contents

- [The Nature of Nature](#) ✓
- [The Nature of Forces](#) ✓
- [The Nature of Waves](#)
- [The Nature of Rotation](#)
- [The Nature of Energy](#)
- [The Nature of Electricity](#)
- [The Nature of Magnetism](#) ✓
- [The Nature of Potential](#) ✓
- [The Nature of Radiation](#)
- [The Nature of Distributions](#) ✓

# The pedagogy predicament.

Top-Down Physics  
Top-Down Physics

- *Historical Approach* (13 – 22 Credits)
  - Mathematical Methods in Physics (0 – 3 Credits)
  - Classical Mechanics (3 Credits)
  - Electricity and Magnetism (3 – 6 Credits)
  - Modern Physics (4 Credits w/ lab)
  - Quantum Mechanics (3 Credits)
  - Thermodynamics and Statistical Mechanics (0 – 3 Credits)
- **Suggested Approach** (19 Credits)
  - Applied Physics (6 Credits)
  - Computational Methods (6 Credits)
  - Modern Physics (4 Credits w/ lab)
  - Quantum Mechanics (3 Credits)

# Nature versus nurture?

- *Student intuition is poor*
  - Stress physical significance (units)
  - Spatial and temporal phase

$$[F] = \frac{kg \cdot m}{s^2} = \frac{(kg \cdot m/s)}{s} = \frac{kg}{s} \cdot \frac{m}{s}$$

$$[P] = \frac{J}{s} = N \cdot \frac{m}{s} = \frac{kg}{s} \cdot \left(\frac{m}{s}\right)^2$$

## The Nature of Nature

### + Philosophical Perspective

### - Physical Significance

- + Introduction
- + Application
- + Homework Questions

### - Time as a Parameter

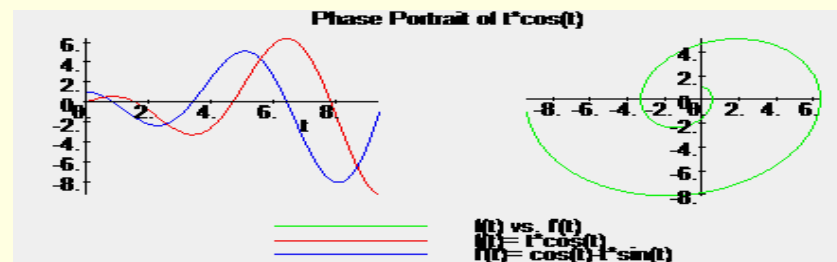
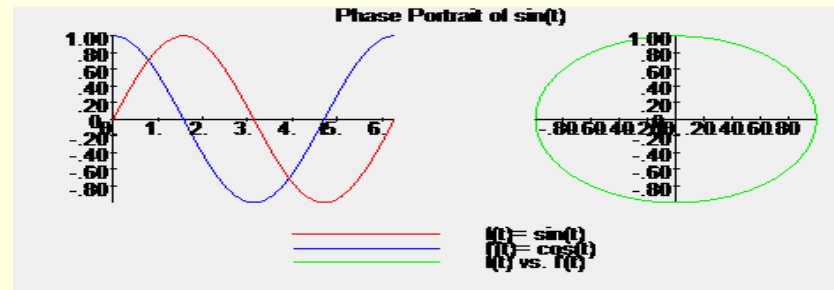
- + Introduction
- + Application
- + Homework Questions

### - Coordinate Systems

- + Introduction
- + Application
- + Homework Questions

### - Vectors and Matrices

- + Introduction
- + Application
- + Homework Questions



# Project based approach.

## ■ Problem

- State a problem of interest

## ■ Analysis

- Provide a start for student

## ■ Conclusion

- Test understanding

## ■ Example: Predator-Prey Model

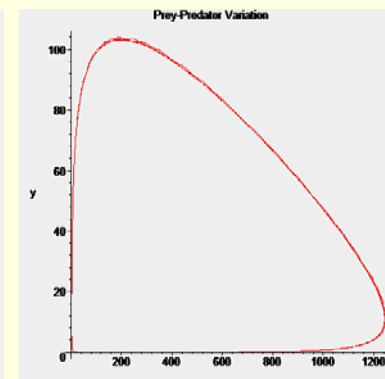
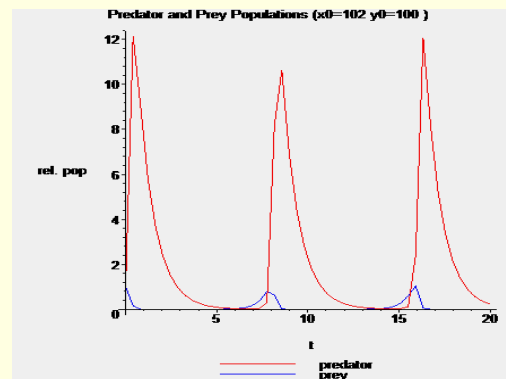
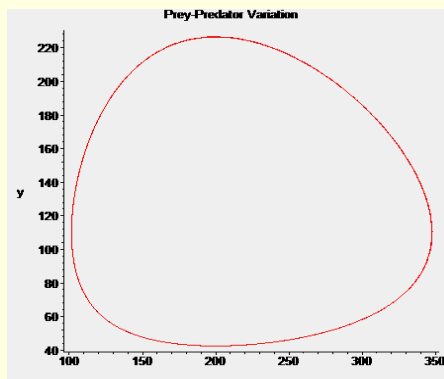
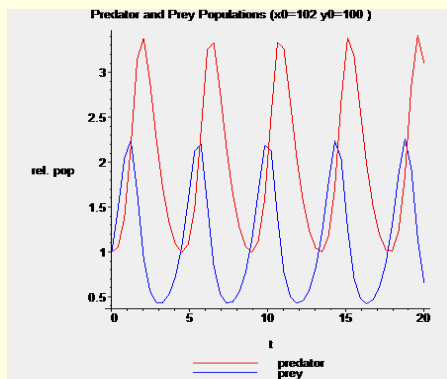
- $k[1]$ ,  $k[3]$  describe the way each behaves independent of the other... when prey is absent predators will perish- hence a negative constant ( $k[1]$ ) is appropriate
- $k[2]$ ,  $k[4]$  describe what happens when the pops. encounter one another... when prey encounter predators they will perish- hence a negative constant ( $k[2]$ ) is appropriate

$$eqpred := \frac{d}{dt} x(t) = k_1 x(t) + k_2 x(t) y(t)$$

$$eqprey := \frac{d}{dt} y(t) = k_3 y(t) + k_4 x(t) y(t)$$

>  $k := [-1.1, 0.01, 2.0, -0.01]$ :

>  $k := [-1.1, 0.1, 2.0, -0.01]$ :



# Astronomy meets CAS!

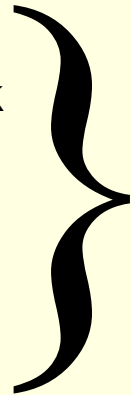
## ■ Central Force Motion

- Start with radial wave equation
- Solve using Maple or Maxima

$$E = \frac{\mu}{2} \left( \frac{\partial}{\partial t} r(\theta) \right)^2 + \frac{L^2}{2\mu r^2} + U(r) \Rightarrow \frac{\partial^2}{\partial \theta^2} \left( \frac{1}{r} \right) + \frac{1}{r} = -\frac{\mu r^2}{L^2} F(r) \Rightarrow$$

```
> findForce:=proc(form::algebraic)
  global r,F;
  # assume r will depend only on theta
  r:=unapply(form,theta);
  diff(1/r(theta),theta$2)+1/r(theta) = -mu/(L^2)*r(theta)^2*f;
  # solve for the force as a function of the angle
  solve(%,f);
  # solve the orbit for theta
  solve(r=r(theta),theta);
  # substitute the angular form into the force solution
  subs(theta=%,%);
  # simplify the force and "build" the force function
  F:=unapply(collect(simplify(%),r),r);
  print('r(theta)='r(theta));
  print('F(r)='F(r));
end proc;
```

- Verify Analytic Work
- Explore Orbits
- Visualize Physics
- Reinforce Concepts



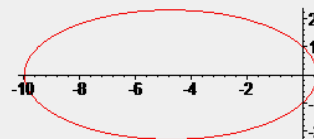
### Elliptic Motion

```
> findForce(alpha/(1+epsilon*cos(theta)));
```

$$r(\theta) = \frac{\alpha}{1 + \epsilon \cos(\theta)}$$

$$F(r) = -\frac{L^2}{\alpha r^2 \mu}$$

```
> alpha:=1: L:=1: mu:=1: epsilon:=0.9;
plot(r(theta), theta=0..2*Pi,
  coords=polar, scaling=constrained);
```

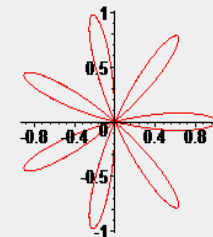


### Cardioid Motion

```
> findForce(B*cos(n*theta));
```

$$r(\theta) = B \cos(n\theta)$$
$$F(r) = \frac{L^2(n^2-1)}{\mu r^3} - \frac{2L^2 n^2 B^2}{\mu r^5}$$

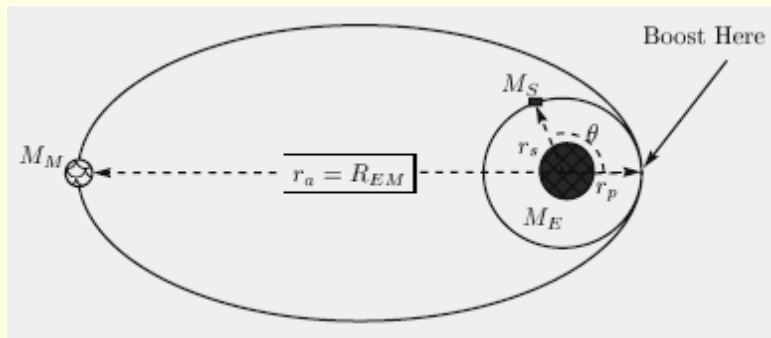
```
> B:=1: n:=7;
plot(r(theta), theta=0..2*Pi,
  coords=polar, scaling=constrained);
```



# Landing on the Moon

## ■ Problem

- If, for an initial Earth orbit of height of 1 km above the surface, a boost in velocity of 40% over the initial velocity is required, then calculate the velocity of the spacecraft at both Perigee and Apogee and determine the eccentricity of the elliptical orbit.



$$(1 - \epsilon)R_{EM} = \alpha \equiv \frac{L^2}{\mu k}$$
$$= \frac{(m_s r_s v_b)^2}{\left(\frac{m_s M_E}{m_s + M_E}\right) (G m_s M_E)}$$

## ■ Conclusion

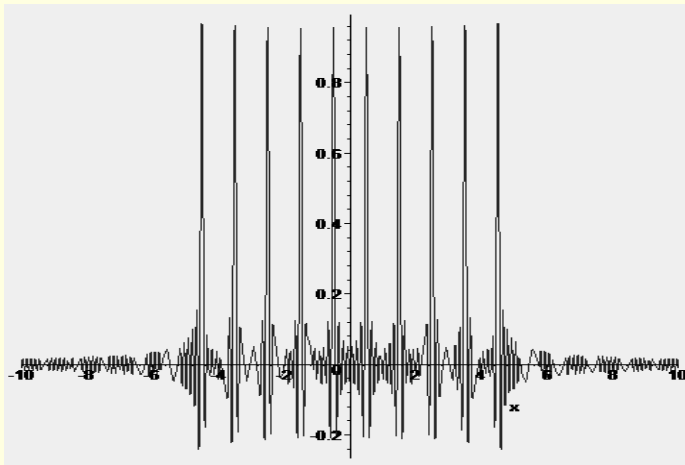
- Derive an expression for the initial velocity of the spacecraft in terms of  $h$ ,  $g$ , and  $R_E$  only.
- Use conservation of angular momentum to determine the velocity of the spacecraft at Apogee.
- Determine the value of the eccentricity of the orbit and plot both the circular and elliptical orbits on the same graph using polar coordinates.

# Real charges and fields.

## ■ Introduction

- A realistic model of a single point charge is given by a sinc function. In one dimension a model of a single particle located at the point,  $x_0$  would take the following form:

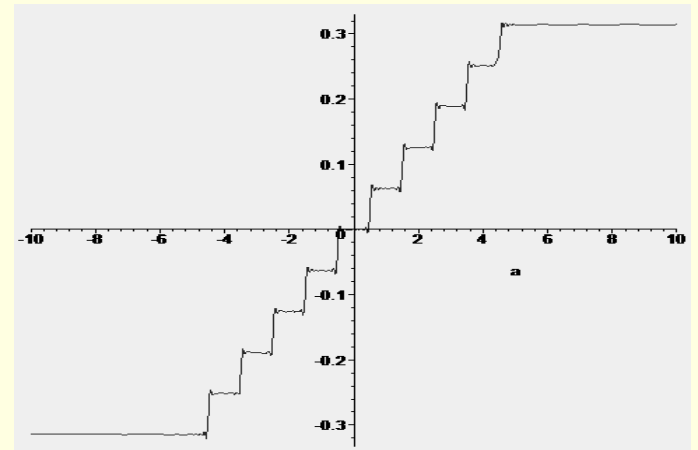
$$\lambda(x) = \lambda_0 \frac{\sin[\alpha(x - x_0)]}{\alpha(x - x_0)}$$



## ■ Problem

- Derive an expression for the electric field due to this charge distribution, and then plot the field...

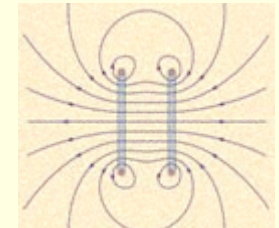
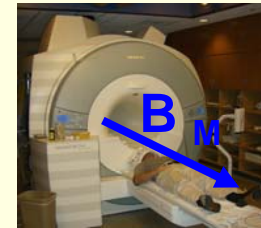
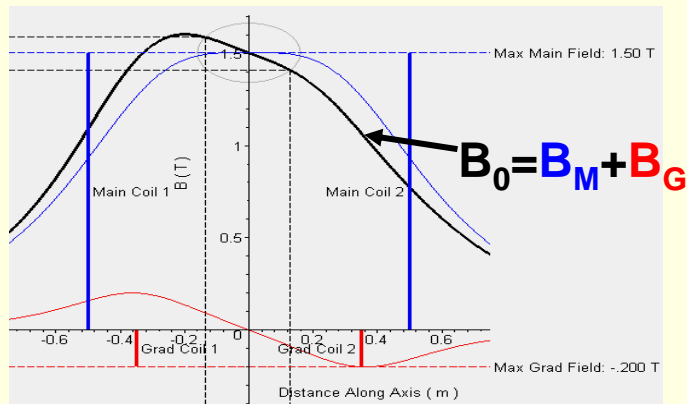
$$\Rightarrow E(a) = \frac{\lambda_0}{\epsilon_0} \sum_{n=0}^N \int_0^a \frac{\sin[\alpha(x_n - x_0)]}{\alpha(x_n - x_0)} dx$$



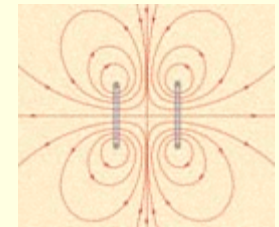
# Mathematics of MRI.

## Basic MR Physics

- Main Field – Helmholtz Coils
- Gradient Field – Maxwell Pair



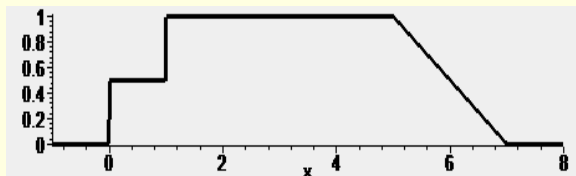
$$B_M = \frac{\mu_0 N_M I_M a^2}{2} \left[ \frac{1}{\left(z - \frac{a}{2}\right)^2 + a^2} + \frac{1}{\left(z + \frac{a}{2}\right)^2 + a^2} \right]$$



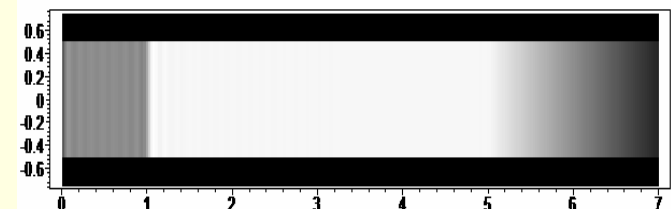
$$B_G = \frac{\mu_0 N_G I_G b^2}{2} \left[ \frac{1}{\left(z - \frac{b}{2}\right)^2 + b^2} - \frac{1}{\left(z + \frac{b}{2}\right)^2 + b^2} \right]$$

## Image Formation

- K-Space concepts – from profile to image



$$S := (g, Gx, t) \rightarrow \int_0^{Lx} m(x) e^{(-i) kx(g, Gx, t) x} dx$$



# Ways of counting...

## ■ Introduction

- The **partition function**,  $Q(N_1, N_2, \dots, N_n)$  gives the number of ways in which the particles may arrange themselves within a given configuration.

$$Q = N! \left( \prod_{n=1}^{\infty} \frac{d_n^{N_n}}{N_n!} \right)$$

- *Distinguishable*

$$Q = \prod_{n=1}^{\infty} \frac{d_n}{N_n! (d_n - N_n)!}$$

- Indistinguishable

- Half-integer spin, *fermions*
- Whole-integer spin, *bosons*

$$Q = \prod_{n=1}^{\infty} \frac{N_n + d_n - 1}{N_n! (d_n - 1)!}$$

## ■ Problem

- How many ways for three particles to partition such that:  $n_A^2 + n_B^2 + n_C^2 = 243$

## ■ Solution

- Three ways: (9,9,9); (3,3,15); (5,7,13)
- Use density of states module to get probability for each type of particle...

$[N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9, N_{10}, N_{11}, N_{12}, N_{13}, N_{14}, N_{15}]$

$$P_{n_c} = \frac{N_n Q_c}{N Q}$$

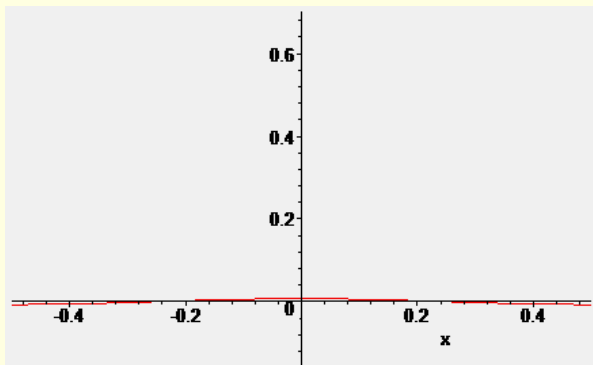
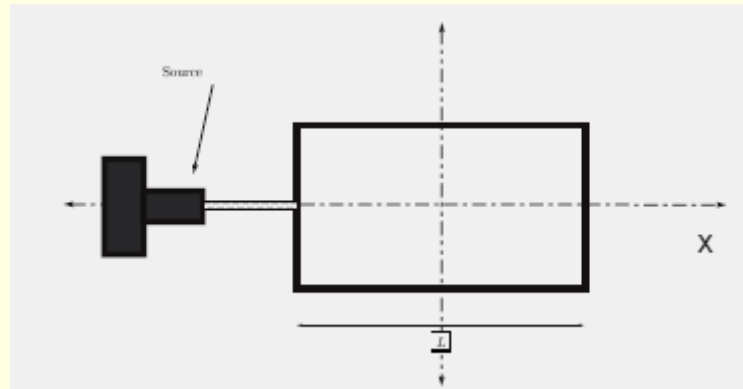
```
> stateProb(3,3,"d");
The distinguishable particle probabilities are:
[0, 0, 1/5, 0, 1/5, 0, 1/5, 0, 1/10, 0, 0, 0, 1/5, 0, 1/10]
> stateProb(3,3,"f");
The fermion particle probabilities are:
[0, 0, 0, 0, 1/3, 0, 1/3, 0, 0, 0, 0, 0, 1/3, 0, 0]
stateProb(3,3,"b");
The boson particle probabilities are:
[0, 0, 2/9, 0, 1/9, 0, 1/9, 0, 1/3, 0, 0, 0, 1/9, 0, 1/9]
```

# Particles from waves?

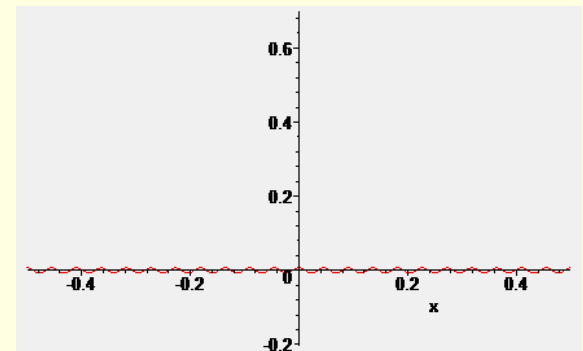
- Put waves in a box in two different ways

- $f[i]:= 1..60$
- $f[i]:= \text{rand}(1..60)$

$$\Psi(x) = \sum_{i=1}^N \frac{A_i}{N} \cos\left(\frac{2\pi f_i}{L} x\right)$$



Which one  
lives longer?



# A work in progress...

## ■ *Material available now!*

- Project descriptions
- Maple solutions
- Related Java applets (Physlets)

## ■ *Coming soon!*

- *Maxima modules*
- *Open Source Physics (OSP) - “Wiki-Text”*

## ■ *Contact Information*

- URL: <http://www.evsis.org>
- Email: [mjs@sc.edu](mailto:mjs@sc.edu)